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ONE-DIMENSION OF NOSTATSIONAL FILTRATION PROCESS OF GAS IN A TWO-LAYER POOR ENVIRONMENT MATHEMATICAL MODEL AND NUMBER OF METHODS OF SOLVING IT

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Abstract– An analysis of the hydrodynamic development of the filtration process of liquids and gases in the porous medium, a mathematical model of the filtration process of gases in dynamically porous media, a finite difference method for solving the boundary value of gas motion in a porous medium and its algorithm considered.

Key words– linear law, quasilinear method, Sweep method

I INTRODUCTION

Based on data on the movement of liquids and gases in the real layer, some complex problems of underground hydraulics are studied and their solutions are found using mathematical methods. The movement of liquid and gas in the natural subsurface begins with the extraction of oil and gas. This movement is completely different from the movement of a pipe or an open well with its own characteristics.

At present, the design and operation of new oil and gas fields and the operation of wells cannot be imagined without the application of the laws of underground hydraulics. How to place the wells in a given layer; how many wells are in the stratum and in what order they should be included; what mode of operation should be maintained in them; how much water needs to be pumped into the layer to maintain pressure; it is necessary to direct and adjust the movement of liquid or gas in the formation, and many such questions are solved on the basis of the laws of underground hydraulics.

If we look at the history of underground hydraulics, the development of this science was founded in the middle of the XIX century by the French engineer G. Darcy. In 1856, he studied filtration phenomena experimentally and developed his own Darcy Law.

Slixter contributed to the development of underground hydraulics. He introduced the concepts of ideal and fictitious (artificial) soil, showing that the porosity of fictitious soil does not depend on the diameter of the particles of volumetric and surface voids, but on their location in the surface.

L.S. Leibenzon was the first to develop a differential equation of gas and gas motion in a porous medium based on theoretical and experimental research. He made a mathematical analysis of the methods of calculating oil and gas reserves in the strata, the problems of oil and gas extraction by water.

Further development of oil and gas underground hydromechanics acad. L.S. Leibenzon's students contributed. In the development of the theory of filtration in the oil and gas aquifer, Acad. S.A. Christianovich, professorial- B.B.Lapuk, I.A.Charny, V.N.Shelka - chev, K.S Basniyev. G.B. The Pikhachevs made a significant contribution.

II MAIN PART

Knowledge of their properties in porous or cracked environments is essential for the efficient operation of oil and gas fields. We consider the problem of non-stationary filtration of gas in two interconnected layers in a porous medium.

The following non-stationary filtration processes are assumed to be:

- Gas layers consist of inhomogeneous porous media;
- The lengths of the gas layers are the same distance;
- In the initial state, the layers are at the same pressure;
- The motion of a gas in both layers obeys the linear law of Darcy;

- The properties of the gas in both layers do not change over time.

Based on these requirements, it is necessary to determine the change in pressure function in both gas layers over time. At this time, the flow rate of gas wells varies, and they can be located anywhere in the gas layer (Fig.1).

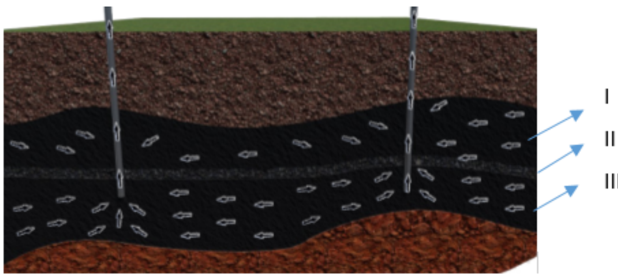

Fig. 1: Gas layers with poor permeability.
I - the first gas layer; II – poorly conductive layer III – the second gas extraction layer.

In the design and analysis of the operation of multilayer gas fields in the filtration processes of gases in a porous environment, it is necessary to take into account the presence of hydrodynamic connections between the layers. In this case, the use of highly efficient methods in the mathematical modeling of the gas filtration process in two-layer systems is required, because the corresponding system of equations is nonlinear and cannot be solved by analytical methods. If both layers are the same in collector properties, the problem can be expressed in a one-dimensional or two-dimensional boundary matter. In this case, the mathematical model of the one-dimensional mass can be thought as a system of equations of the nonlinear parabolic type.(1)

$$\begin{cases} \frac{\partial}{\partial x}\left[k_1(x)\frac{\partial P_1^2}{\partial x}\right] = 2\mu a_1 m_1 \frac{\partial P_1}{\partial t} - \frac{k_\Pi}{h_\Pi h_1}(P_2^2 - P_1^2), \\ \frac{\partial}{\partial x}\left[k_2(x)\frac{\partial P_2^2}{\partial x}\right] = 2\mu a_2 m_2 \frac{\partial P_2}{\partial t} + \frac{k_\Pi}{h_\Pi h_1}(P_2^2 - P_1^2) - Q, \end{cases}$$

$$0 < x < L \qquad (1.1)$$

Initial and boundary conditions

$$P_1(x) = P_{1H}(x) \qquad P_2(x) = P_{2H}(x) \qquad at \qquad t = 0 \qquad (1.2)$$

$$-k_1 h_1 \frac{\partial P_1}{\partial x} = \alpha(P_A - P_1); \quad -k_2 h_2 \frac{\partial P_2}{\partial x} = \alpha(P_A - P_2) \qquad x = 0 \qquad (1.3)$$

$$-k_1 h_1 \frac{\partial P_1}{\partial x} = \alpha(P_B - P_1); \quad -k_2 h_2 \frac{\partial P_2}{\partial x} = \alpha(P_B - P_2) \qquad x = L \qquad (1.4)$$

$$\int_S \frac{k_i h_i}{\mu}\frac{\partial P_i}{\partial n} ds = -q_i(t); \qquad i = 1,2 \qquad (1.5)$$

The following symbols are accepted in the system of equations and boundary conditions:

- P_1, P_2 - lower and upper layer pressures, respectively;
- P_H - initial layer pressure;
- k_1 and k_2 – layer permeability in the lower and upper layers, respectively;
- k_P – permeability coefficient of weak conductivity layer;
- h_1 va h_2 - the thickness of the bottom and top layer, respectively;
- h_P - weakly conductive layer thickness;
- μ – gas viscosity coefficient;
- $q(t)$ – bottom layer well flow;
- m_1, m_2 – porosity of the lower and upper layers, respectively;
- s_i – well contour;
- n_1 - number of wells.

To include dimensionless variables in the system of equations and in boundary conditions, we include the following.

$$x^* = \frac{x}{L}; \qquad k_1^* = \frac{k_1}{k_x}; \qquad k_2^* = \frac{k_2}{k_x}; \qquad k_P^*(x) = \frac{k_P}{k_x};$$

$$P_1^* = \frac{P_1}{P_x}; \qquad P_2^* = \frac{P_2}{P_x}; \qquad \tau = \frac{k_x P_x t}{\mu L^2}.$$

By performing these changes in the system and dropping the asterisk for convenience, we come to the following dimensionless problem:

$$\begin{cases} \frac{\partial}{\partial x}\left[k_1(x)\frac{\partial P_1^2}{\partial x}\right] = 2\alpha_1 m_1 \frac{\partial P_1}{\partial t} - \frac{k_p}{h_p}\frac{L^2}{h_1}(P_2^2 - P_1^2), \\ \frac{\partial}{\partial x}\left[k_2(x)\frac{\partial P_2^2}{\partial x}\right] = 2\alpha_2 m_2 \frac{\partial P_2}{\partial t} + \frac{k_p}{h_p}\frac{L^2}{h_2}(P_2^2 - P_1^2) - \delta q(t), \end{cases}$$

$$0 < x < 1 \qquad (1.6)$$

Initial and boundary conditions

$$P_1(x) = P_{1H}(x) \qquad P_2(x) = P_{2H}(x) \qquad at \qquad t = 0 \qquad (1.7)$$

$$-k_1 h_1 \frac{\partial P_1}{\partial x} = \alpha(P_A - P_1); \quad -k_2 h_2 \frac{\partial P_2}{\partial x} = \alpha(P_A - P_2) \qquad x = 0 \qquad (1.8)$$

$$-k_1 h_1 \frac{\partial P_1}{\partial x} = \alpha(P_B - P_1); \quad -k_2 h_2 \frac{\partial P_2}{\partial x} = \alpha(P_B - P_2) \qquad x = 1 \qquad (1.9)$$

$$\int_S \frac{k_i h_i}{\mu}\frac{\partial P_i}{\partial n} ds = -q_i(t); \qquad i = 1,2 \qquad (1.10)$$

Limited difference method and its algorithm for solving the boundary value problem of gas motion in a two-layer porous medium

We use the non-disclosed finite difference method to solve the dimensional boundary value problem above (1.6) - (1.10) (7).

To solve the problem using this numerical method, we construct a grid of equal steps in the field

$$\{0 < x < 1, \quad 0 \le t \le 1\}$$

.

$$w_{h\tau} = \left\{ x_i = ih, \quad i = 0,1,...,n, \quad h = \frac{1}{n}, \quad t_j = j\tau, \quad j = 0,1,... \right\}$$

and by approximating the system of equations in the grid, we obtain the following finite difference scheme (1).

$$k_{1i-0.5}P_{1i-1}^2 - (k_{1i-0.5} + k_{1i+0.5})P_{1i}^2 + k_{1i+0.5}P_{1i+1}^2 - \frac{h^2}{\tau}2\alpha_1 m_1(P_{1i} - \hat{P_{1i}}) + \frac{h^2 k_{Pi}}{h_P}\frac{L^2}{h_1}(P_{2i}^2 - P_{1i}^2) = 0;$$

$$k_{2i-0.5}P_{2i-1}^2 - (k_{2i-0.5} + k_{2i+0.5})P_{2i}^2 + k_{2i+0.5}P_{2i+1}^2 - \frac{h^2}{\tau}2\alpha_2 m_2(P_{2i} - \hat{P_{2i}}) - \frac{h^2 k_{Pi}}{h_P i}\frac{L^2}{h_2}(P_{2i}^2 - P_{1i}^2) = 0;$$

The obtained differential equations are not linear with respect to the pressure function P, so the iteration method based on the quasilinear functions of nonlinear method is used. According to these methods, the nonlinear members of finite difference equations are presented as follows:

$$\varphi(P) = \varphi(\tilde{P}) + (P - \tilde{P})\frac{\partial \varphi(\tilde{P})}{\partial P}. \qquad (1.11)$$

Here, P is the approximate value of the $\tilde{P}$ function determined during the iteration process

$$\tilde{P} = P_i^{(S)}, \quad and \quad P_i^{(0)} = \hat{P}_i.$$

The iteration process continues until the following conditions are met

$$max_{i,j}\left|P_{1i}^{(s)} - P_{1i}^{(s-1)}\right| \le \varepsilon, \quad max_{i,j}\left|P_{2i}^{(s)} - P_{2i}^{(s-1)}\right| \le \varepsilon. \qquad (1.12)$$

Here:
ε – iteration accuracy, a small amount known in advance;
s - number of iterations.

If formula (1.11) is written for a nonlinear pressure function, we have the following formula

$$P^2 \approx 2\tilde{P}P - \tilde{P}^2.$$

Then, after applying the method of quasilinear functions of nonlinear terms, the coefficients of these quasilinear differential equations are as follows:

$$a_i = 2\tilde{P}_{1i-1}k_{1i-0.5}; \quad c_i = 2\tilde{P}_{1i+1}k_{1i+0.5};$$

$$b_i = a_i + c_i + \frac{h^2}{\tau} 2\alpha_1 m_1 + \frac{h^2 k_{\Pi i}}{h_\Pi} \frac{L^2}{h_1};$$

$$d_i = \frac{h^2 k_{\Pi i}}{h_\Pi} \frac{L^2}{h_1};$$

$$f_i = \frac{h^2}{\tau} 2\alpha_1 m_1 \hat{P}_{1i} + k_{1i-0.5} \tilde{P}^2_{1i-1} -$$
$$-(k_{1i-0.5} + k_{1i+0.5}) \tilde{P}^2_{1i} + k_{1i+0.5} \tilde{P}^2_{1i+1};$$

$$a'_i = 2\tilde{P}_{2i-1} k_{2i-0.5}; \quad c'_i = 2\tilde{P}_{2i+1} k_{2i+0.5};$$

$$b'_i = a'_i + c'_i + \frac{h^2}{\tau} 2\alpha_2 m_2 + \frac{h^2 k_{\Pi i}}{h_\Pi} \frac{L^2}{h_2};$$

$$d'_i = \frac{h^2 k_{\Pi i}}{h_\Pi} \frac{L^2}{h_2};$$

$$f'_i = \frac{h^2}{\tau} 2\alpha_2 m_1 \hat{P}_{2i} + k_{2i-0.5} \tilde{P}^2_{2i-1} -$$
$$-(k_{2i-0.5} + k_{2i+0.5}) \tilde{P}^2_{2i} + k_{2i+0.5} \tilde{P}^2_{2i+1};$$

We use the sweep method to solve this finite separation system. Then we have this system of finite differences above and the system of finite differences below the boundary conditions.

$$a_i P_{1i-1} - b_i P_{1i} + c_i P_{1i+1} + d_i P_{2i} = -f_i; \tag{1.13}$$

$$(3k_{10}h_1 - 2h\lambda\alpha) P_{10} - 4k_{11} h_1 P_{11} + k_{12} h_1 P_{12} = 2h\lambda\alpha P_A \tag{1.14}$$

$$(3k_{1n}h_1 - 2h\lambda\alpha) P_{1n} + 4k_{1n-1} h_1 P_{1n-1} - k_{1n-2} h_1 P_{1n-2} = -2h\lambda\alpha P_A \tag{1.15}$$

$$a'_i P_{2i-1} - b'_i P_{2i} + c'_i P_{2i+1} + d' P_{1i} = -f'_i; \tag{1.16}$$

$$(3k_{20}h_2 - 2h\lambda\alpha) P_{20} - 4k_{21} h_2 P_{21} + k_2 h_2 P_{22} = 2h\lambda\alpha P_A \tag{1.17}$$

$$(3k_{2n}h_2 - 2h\lambda\alpha) P_{2n} + 4k_{2n-1} h_2 P_{2n-1} - k_2 h_{2n-2} P_{2n-2} = -2h\lambda\alpha P_A \tag{1.18}$$

$$i, j = 1, 2, ..., N-1.$$

The solution of this finite distribution system (1.13) - (1.18) is determined from the following formulas

$$P_{1i} = A_i P_{1i+1} + B_i P_{2i+1} + C_i \tag{1.19}$$

$$P_{2i} = A'_i P_{2i+1} + B'_i P_{1i+1} + C'_i \tag{1.20}$$

$$i = 1, 2, ..., n-1.$$

Here:

$$A_i = \frac{c_i (b'_i - a'_i A'_{i-1})}{R_i}; \quad B_i = \frac{c'_i (a_i B_{i-1} + d_i)}{R_i}; \tag{1.21}$$

$$A'_i = \frac{c'_i (b_i - a_i A_{i-1})}{R_i}; \quad B'_i = \frac{c_i (a'_i B'_{i-1} + d'_i)}{R_i}; \tag{1.22}$$

$$C_i = \frac{(a_i B_{i-1} + d_i)(a'_i C'_{i-1} + f'_i) + (a_i C_{i-1} + f_i)(b'_i - a' A'_{i-1})}{R_i}; \tag{1.23}$$

$$C'_i = \frac{(a'_i B'_{i-1} + d'_i)(a_i C_{i-1} + f_i) + (a'_i C'_{i-1} + f'_i)(b_i - a_i A_{i-1})}{R_i}; \tag{1.24}$$

$$R_i = (b_i - a_i A_{i-1})(b'_i - a'_i A'_{i-1}) - (a_i B_{i-1} + d_i)(a'_i B'_{i-1} + d').$$

$$i = 1, 2, ..., n-1.$$

Here $A_0; B_0; C_0; A_0'; B_0'; C_0'$; s values are determined from the boundary conditions

$$A_0 = \frac{(b_1 - 4c_1)k_{11}h_1}{a_1k_{12}h_1 - (3k_{10}h_1 - 2h\alpha)c_1}; \qquad (1.25)$$

$$B_0 = -\frac{d_1k_{12}h_1}{a_1k_{12}h_1 - (3k_{10}h_1 - 2h\alpha)c_1}; \qquad (1.26)$$

$$C_0 = \frac{f_1k_{12}h_1 + 2h\alpha c_1}{a_1k_{12}h_1 - (3k_{10}h_1 - 2h\alpha)c_1}; \qquad (1.27)$$

$$A_0' = \frac{(b_1' - 4c_1')k_{21}h_2}{a_1'k_2h_{22} - (3k_{20}h_2 - 2h\alpha)c_1'}; \qquad (1.28)$$

$$B_0' = -\frac{d_1'k_{22}h_2}{a_1'k_{22}h_2 - (3k_{20}h_2 - 2h\alpha)c_1'}; \qquad (1.29)$$

$$C_0' = \frac{f_1'k_{22}h_2 + 2h\alpha c_1'}{a_1'k_{22}h_2 - (3k_{20}h_2 - 2h\alpha)c_1'}. \qquad (1.30)$$

using formulas (1.13 and (1.16) (for $i = n-1$), the right-hand boundary conditions (1.15) and (1.18) and the formulas (1.19), (1.20) (for $i = n-1$) on the ng side P_{1n} va P_{2n} we find.

After the reorganization, we obtain the following systems of equations, two of which are unknown P_{1n} and P_{2n}:

$$\left[(3a_{n-1} - c_{n-1}) - (4a_{n-1} - b_{n-1})A_{n-1} - d_{n-1}B_{n-1}'\right]P_{1n} + \\ + \left[(4a_{n-1} - b_{n-1})B_{n-1} - d_{n-1}A_{n-1}'\right]P_{2n} = \\ = \left[d_{n-1}C_{n-1}' + f_{n-1} + (4a_{n-1} - b_{n-1})\right];$$

$$\left[(3a_{n-1}' - c_{n-1}') - (4a_{n-1}' - b_{n-1}')A_{n-1}' - d_{n-1}'B_{n-1}\right]P_{2n} + \\ + \left[(4a_{n-1}' - b_{n-1}')B_{n-1}' - d_{n-1}'A_{n-1}\right]P_{1n} = \\ = \left[d_{n-1}'C_{n-1} + f_{n-1} - (4a_{n-1}' - b_{n-1}')\right].$$

From these systems P_{1n} and P_{2n}, Depending on the solution we get:

$$P_{1n} = (S_2 \cdot S_3' - S_3 \cdot S_1')/(S_1 \cdot S_1' - S_2 \cdot S_2'); \qquad (1.31)$$

$$P_{2n} = (S_3 \cdot S_2' - S_1 \cdot S_3')/(S_1 \cdot S_1' - S_2 \cdot S_2'); \qquad (1.32)$$

Here:

$$S_1 = \left[(3a_{n-1} - c_{n-1}) - (4a_{n-1} - b_{n-1})A_{n-1} - d_{n-1}B_{n-1}'\right];$$

$$S_2 = \left[-(4a_{n-1} - b_{n-1})B_{n-1} - d_{n-1}A_{n-1}'\right];$$

$$S_3 = \left[f_{n-1} + d_{n-1}C_{n-1}' + (4a_{n-1} - b_{n-1})C_{n-1}'\right];$$

$$S_1' = \left[(3a_{n-1}' - c_{n-1}') - (4a_{n-1}' - b_{n-1}')A_{n-1}' - d_{n-1}'B_{n-1}\right];$$

$$S_2' = \left[-(4a_{n-1}' - b_{n-1}')B_{n-1}' - d_{n-1}'A_{n-1}\right];$$

$$S_3' = \left[f_{n-1}' + d_{n-1}'C_{n-1} + (4a_{n-1}' - b_{n-1}')C_{n-1}\right].$$

Conducting computer experiments and their analysis

The program was developed based on a mathematical model and a computational algorithm. The software consists of a block of input data, a block of calculation of the main indicators of gas field operation and a block of output of numerical results. The numerical results of the calculated indicators are presented to the user in visual form in tabular and graphical form. Calculation experiments were performed on different values of well debits and formation parameters.

These parameters and their values are given below:

- n = 101 - number of steps for a discrete field;
- h = 0.01 - step;
- p_{n1} = 200 atm. - pressure in the first layer;
- p_{n2} = 200 atm. - pressure in the second layer;
- Lesson k_1 = 0.01 - the coefficient of permeability of the first layer;
- Lesson k_2 = 0.01 - the coefficient of permeability of the second layer;

- Lesson $k_p = 0.0000001$ - sluggish layer permeability coefficients;
- $\mu = 0.02\ s_{pz}$ - gas viscosity coefficient;
- $m = 0.2$ - porosity coefficient;
- $x_l = 10000$ meters - the length of the layer;
- $q = 1000000\ m^3$/day - flow rate of wells;
- $n_t = 720$ days - calculation time;
- $h_1 = 10$ meters - the thickness of the first layer;
- $h_2 = 10$ meters - the thickness of the second layer;
- $h_p = 1$ meter - the thickness of the permeable layer.

Computational experiments were performed mainly on different values of stratum permeability, gas viscosity and well flow rate. Fig. 2 and 3 show the pressure drop in the wells and the pressure changes in the layers as a result of 720 days of operation of the field. There are two wells near the center with a flow rate $q = 1000000\ m^3$/sec. The results in these figures show that the pressure drop in the upper layer at the initial time is very small and the pressure drop stabilizes over time. This process is clearly seen in Fig. 2. The reason for the extraction of gas from the bottom layer is that here the pressure drop in the wells decreases rapidly at the beginning and then stabilizes.

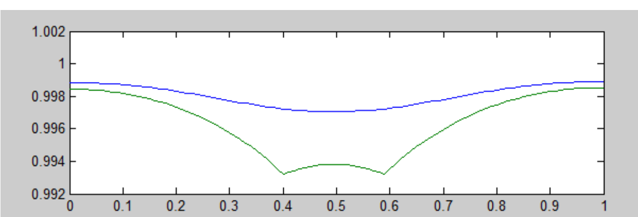

Fig. 2: Graph of pressure changes in the upper and lower layers ($k_p = 0.0000001$ lessons)

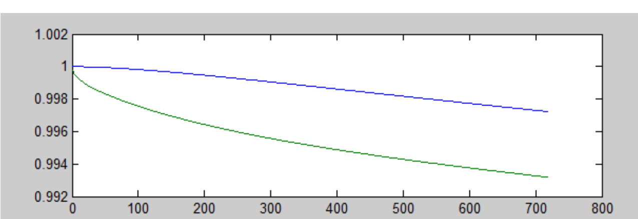

Fig. 3: Graph of pressure drop in the well and at the corresponding point in the upper layer ($k_p = 0.0000001$ lesson)

III CONCLUSION

The methods and techniques developed to calculate the key performance indicators of two-layer gas fields, as well as the software can be used in process analysis and design, as well as in the operation of multi-layer oil and gas fields.

Using the proposed numerical method, the solution of systems of equations can be easily generalized for a system of three or more equations. The results obtained are useful for analyzing the development of multilayer gas fields in the dynamic relationship between layers.

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