*Acta of Turin Polytechnic University in Tashkent*, 2021, 11, 21-24
Published Online June 2021 in Acta TTPU (http://www.acta.polito.uz/)

# Comparative analysis of numerical tools Simulink and Simscape for the analysis of multi-degree of freedom mass-spring-damper systems.

**Seyran Asanov**
Turin Polytechnic University in Tashkent,17, Little Ring Road street, Tashkent, Uzbekistan
Email: seyran.asanov@polito.uz

**Abstract**– This paper describes the solution of the system of linear differential equations of second order with constant coefficients which describe the motion of two-degree of freedom mass-spring-damper system with a forced excitation using the MATLAB tools Simulink and Simscape. The results show that both models show quite similar results. The obvious advantage of Simulink model is that the engineer has full control over the differential equation he is to implement in the model whereas the Simscape model provides a ready solution in the form of "physical" blocks that represent real physical components.

**Key words**– Lagrange equations, mass-spring-damper system, Simulink, Simscape

## I Introduction

In this paper is presented a problem of 2-degree of freedom mass-spring damper system with an input force applied to the second mass. The model of the system under investigation is presented in Figure 1. The strategy of the analysis starts with the elaboration of the mathematical model by means of Lagrange's equations of the second kind applied to the system. Afterwards, a Simulink model will be proposed in order to solve numerically the analytical equations. Also, a Simscape model will be represented a possible alternative of the Simulink model. Later on, the results of both models will be compared. It is noteworthy that for the scope of the paper a sinusoidal input force $F = F_0 sin(\omega t)$.Besides, the damping coefficients for the two dampers involved in the model is assumed to be constant.The parameters of the problem are presented in Table 1. The initial conditions of the system are set to $x(0) = 0$ and $\dot{x}(0) = 0$.

| System parameters | Value | Unit of measurement |
|---|---|---|
| Mass $m_1$ | 70 | kg |
| Mass $m_2$ | 140 | kg |
| Spring stiffness $k_1$ | 500 | $\frac{N}{m}$ |
| Spring stiffness $k_2$ | 250 | $\frac{N}{m}$ |
| Damping coefficient $c_1$ | 10 | $\frac{Ns}{m}$ |
| Damping coefficient $c_2$ | 50 | $\frac{Ns}{m}$ |
| Force amplitude $F_0$ | 100 | N |
| Frequency $\omega$ | 4 | $\frac{rad}{s}$ |

**Table 1:** Model parameters

## II Methodology of developing a mathematical model

An important step in the solution of the problem is to develop a mathematical model describing the motion of the system under investigation.To do this we determine the equations of motion of the above-mentioned system using Lagrange's equations of the second kind[1].The most general form of the Lagrange equations are represented by[2, 3]

$$\frac{d}{dt}(\frac{\partial E_k}{\partial \dot{q}_k}) - \frac{\partial E_k}{\partial q_k} + \frac{\partial E_p}{\partial q_k} = Q_{b_k}, k = 1,2,....n \qquad (1)$$

Where $q_k$ are generalized coordinates, $E_k$- kinetic energy of the system, $E_p$- potential energy of the system and $Q_{b_k}$-generalized force.
In order to obtain the required equations, we need to determine kinetic and potential energies of the system.
If we define the generalized coordinates as

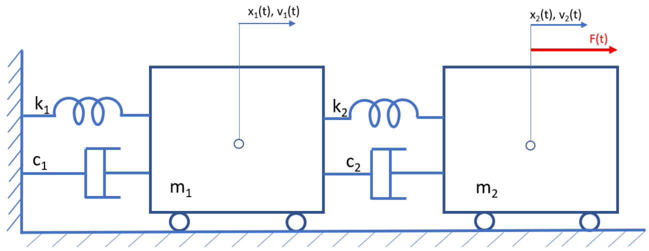


**Fig. 1:** 2 degree of freedom mass-spring-damper system representation

$$q_1 = x_1 \tag{2}$$

$$q_2 = x_2 \tag{3}$$

And the generalized velocities as

$$q_1 = \dot{x}_1 \tag{4}$$

$$q_2 = \dot{x}_2 \tag{5}$$

Kinetic energy of the system is equal to sum of the kinetic energies of the masses $m_1$ and $m_2$ :

$$E_k = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 \tag{6}$$

The potential energy of the system is equal to the sum of the potential energies of the springs $k_1$ and $k_2$:

$$E_p = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_2 - x_1)^2 \tag{7}$$

Notice that the second spring has both ends in motion. For this reason the relative extension/compression $(x_2 - x_1)$ has to be taken into account.

Since dampers are also present in the system under investigation, a term characterizing the energy dissipation has to be introduced. For this purpose we use the Rayleigh dissipate function.The Rayleigh dissipate function can be expressed in the following way in terms the damping coefficients $c_1$ and $c_2$ and the speeds of the masses $\dot{x}_1$, $\dot{x}_2$:

$$D = \frac{1}{2} c_1 \dot{x}_1^2 + \frac{1}{2} c_2 (\dot{x}_2 - \dot{x}_1)^2 \tag{8}$$

Similarly to the case of the potential energy, the second contribution of the Rayleigh dissipate function depends on the relative speed of the damper $(\dot{x}_2 - \dot{x}_1)$ because both ends of the second damper are in motion.

The generalized forces also have to be taken into consideration. In the particular case of the problem under investigation, only the generalized force related to the second mass is non-zero since an input force is applied to mass $m_2$:

$$Q_{b_1} = 0 \tag{9}$$

$$Q_{b_2} = F(t) \tag{10}$$

Substituting the generalized coordinates, the potential energy of the system can be expresses as:

$$E_p = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2 - k_2 x_2 x_1 + \frac{1}{2} k_2 x_1^2 \tag{11}$$

At the same time, the Rayleigh dissipate fucntion can be elaborated as

$$D = \frac{1}{2} c_1 \dot{x}_1^2 + \frac{1}{2} c_2 \dot{x}_2^2 - c_2 \dot{x}_2 \dot{x}_1 + \frac{1}{2} c_2 \dot{x}_1^2 \tag{12}$$

Taking into account the arrangement of the generalized coordinates and the dissipate function, Lagrange's equations of the second kind for the mechanical system in Figure 1 are:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}_1}\right) - \frac{\partial E_k}{\partial x_1} = -\frac{\partial E_p}{\partial x_1} - \frac{\partial D}{\partial_1} + Q_{b_1} \tag{13}$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}_2}\right) - \frac{\partial E_k}{\partial x_2} = -\frac{\partial E_p}{\partial x_2} - \frac{\partial D}{\partial x_2} + Q_{b_2} \tag{14}$$

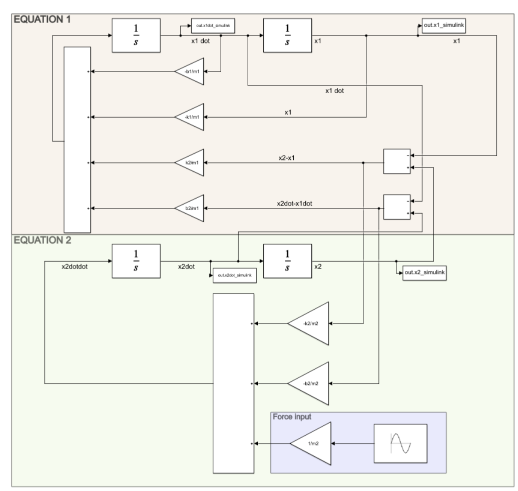


**Fig. 2:** Simulink model

After computing the partial derivatives of the kinetic and potential energies as well the Rayleigh dissipate functions and substituting them into the Lagrange's equations, we are able to obtain a system of equations of motion:

$$\ddot{x}_1 = -\frac{k_1}{m_1} x_1 + \frac{k_2}{m_1}(x_2 - x_1) - \frac{b_1}{m_1}\dot{x}_1 + \frac{b_2}{m_1}(\dot{x}_2 - \dot{x}_1) \tag{15}$$

$$\ddot{x}_2 = -\frac{k_2}{m_2}(x_2 - x_1) - \frac{b_2}{m_2}(\dot{x}_2 - \dot{x}_1) + \frac{F(t)}{m_2} \tag{16}$$

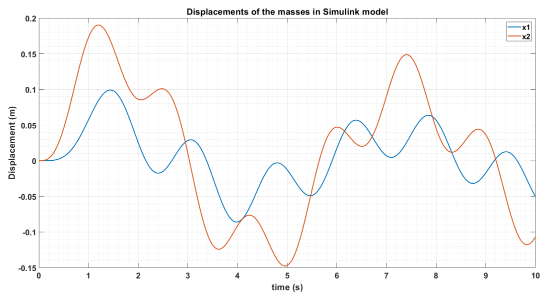


**Fig. 3:** Displacement of masses in Simulink model

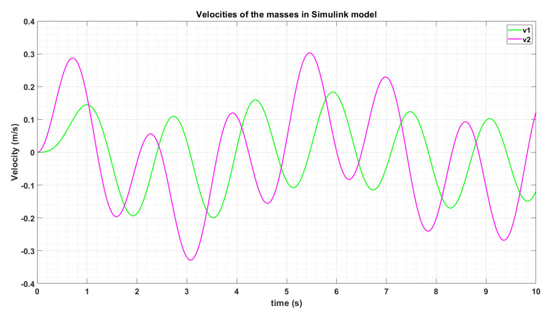


**Fig. 4:** Velocities of masses in Simulink model

Where $\dot{x_1}$ and $\dot{x_2}$ are velocities of the masses $m_1$ and $m_2$ respectively, whereas $\ddot{x_1}$ and $\ddot{x_2}$ are their corresponding accelerations.

Thus, the system to be solved consists of second order linear differential equations. Masses, spring stiffnesses and damping coefficients are assumed to be constant. In this regard, we are to solve the system of second order differential equations with constant coefficients. The developed mathematical model will be solved using Simulink in Chapter 3 and with the help of Simscape in Chapter 4

## III Simulink model

As was discussed in the previous chapter, the non-homogeneous system of second order linear differential equations with constant coefficients is proposed to be solved using the numerical facility of MathWorks called Simulink. In general, Simulink[4] is a very powerful tool for creating mathematical models using different blocks which perform a certain function. Connecting the blocks with lines in an appropriate manner, we create the signal path between several blocks. The Simulink model of the equations developed in Chapter 2 is represented in Figure 2. As can be seen in the figure, both equations in the model are interlinked because they have some common terms, and these interlinks are expressed by the signal lines present in both the equation 1 are and equation 2 are. Moreover, the force input is related to the mass m2 and it is also highlighted with a different color in the model. Simulating the proposed model, the results on displacements of the masses (Figure 3) and their corresponding velocities (Figure 4) were obtained.

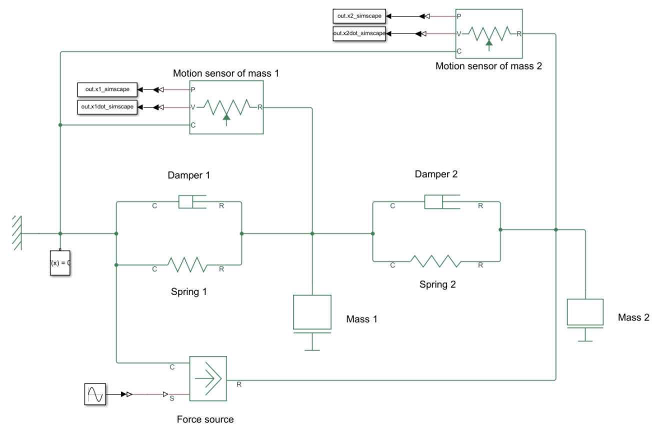


**Fig. 5:** Simscape model

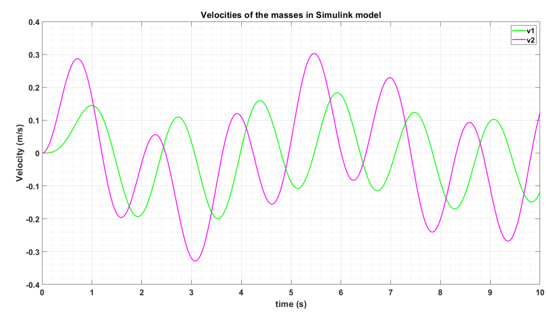


**Fig. 6:** Displacements of the masses in Simscape model

## IV Simscape model

Simscape[5] is a built-in tool of Simulink that allows to create “physical” models of different systems. That is, instead of equations and mathematical blocks, blocks representing the physical objects can be introduced. Every “physical” block of Simscape contains inside the mathematical relationships describing the system. The Simscape model proposed for the article is presented in Figure 5.

In the model proposed, all the springs, masses and dampers are represented by separate blocks. Moreover, a block representing the force input and two motion sensor blocks have to introduced. The former serves as an ideal force source, where as the motion sensor blocks allow to measure the displacements and velocities of the masses.It is very important to note that one port of both the motion sensors has to be connected to translational fixed reference frame. This will allow to measure the displacements and

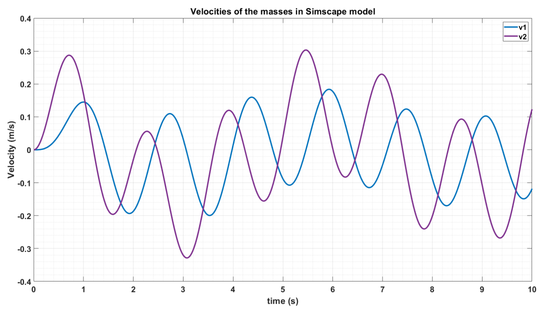


**Fig. 7:** Velocities of the masses in Simscape model

velocities with respect to the ground. In analogy with the Simulink model, the Simscape model was simulated and the corresponding results on displacements (Figure 6) and velocities (Figure 7) were obtained.

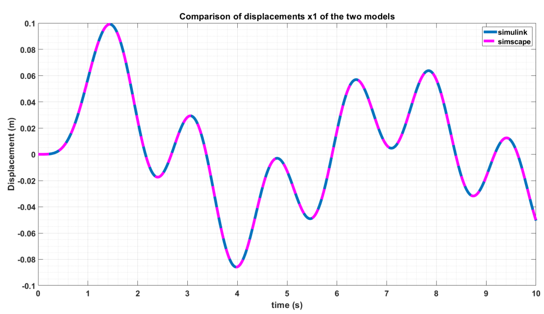


**Fig. 8:** Comparison of displacements $x_1$ obtained from Simulink and Simscape models

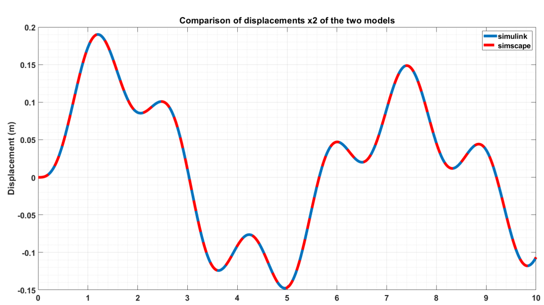


**Fig. 9:** Comparison of displacements $x_2$ obtained from Simulink and Simscape modelsl

## V Conclusion

The obtained results are very close to those of the Simulink model. Therefore, their corresponding comparison can be provided. For the purpose of comparison, displacements of the masses have been taken into consideration. As is represented in Figure 8, both models show more or less identical results in terms of displacements $x_1$ of the mass $m_1$. Quite analogically, the displacements s $x_2$ of the mass $m_2$ obtained from the proposed two model (Figure 9) are also almost identical.

In this paper two different approaches to model multi-degree of freedom mass-spring-damper systems have been demonstrated. The former is related to constructing a mathematical model using the elaborated system of linear differential equations whereas the latter one is more related to physical modeling of mechanical systems by means of special purpose blocks. In general, the results show that both the model solve well the system of linear equations. The advantage of Simulink model is that an engineer has a full control of the model he is creating while the Simscape model contains mathematical relationships inside every "physical block".

.

## References

[1] Peter Sivák and Darina Hroncová. State-space model of a mechanical system in matlab/simulink. *Procedia Engineering*, 48:629–635, 2012. Modelling of Mechanical and Mechatronics Systems.

[2] L Author Meirovitch and RG Reviewer Parker. Fundamentals of Vibrations. *Applied Mechanics Reviews*, 54(6):B100–B101, 11 2001.

[3] Giancarlo Genta. *Vibration Dynamics and Control*. Springer US, 2009.

[4] Design models of dynamic systems in simulink. `https://www.mathworks.com/help/matlab/language-fundamentals.html`.

[5] Model and simulate multidomain physical systems in simscape. `https://www.mathworks.com/help/physmod/simscape/`.